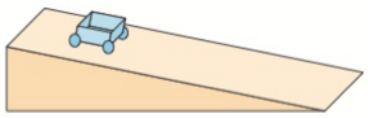
Topics in Arithmetic **Upper and Lower Bounds** DATE:_____

NAME:_____

Example: In a science experiment, a trolley travelled 5.6 meters in 3.44 seconds. Calculate the average speed of the trolley.



Measuring instrument

Work out the upper bound for the speed. Round to 3 s.f.

Work out the lower bound for the speed. Round to 3 s.f.

When carrying out calculations with numbers that have been rounded it is possible to calculate the range of answers that can be produced depending on the accuracy to which the numbers have been given.

If a number has been written as 5.6 correct to one decimal place then the true value of the number lies between ☐ and ☐ The value at the lower boundary of the interval is called the **lower bound**. The value at the upper boundary is called the **upper bound**. In this case, ☐ is the lower bound and ☐ is the upper bound.

If a number has been written as 3.44 correct to 3 significant figures, then the true value of the number lies between ☐ to ☐ so the lower bound is ☐ and the upper bound is ☐

Example 6

Write down **i** the lower bound **ii** the upper bound of

a 64 correct to 2 significant figures **b** 32.8 correct to 1 decimal place.

Solution 6

a **i** lower bound = ☐

ii upper bound = ☐

63 63.5 64 64.5 65

Numbers in the red interval from 63.5 to 64.5 are closer to 64 than to 63 or to 65 The lower bound (the value at the lower boundary of this interval) is 63.5 The upper bound (the value at the upper boundary of the interval) is 64.5

b **i** lower bound = ☐

ii upper bound = ☐

32.7 32.75 32.8 32.85 32.9

Numbers in the red interval from 32.75 to 32.85 are closer to 32.8 than to 32.7 or to 32.9

If numbers in calculations have been rounded, the final answer will not be exact. It is, however, possible to find the lower bound and the upper bound of the answer.

For example, in the product 4.5×6.4 where both numbers have been rounded to 1 decimal place

to find the upper bound of the product, work out $4.55 \times 6.45 = 29.3475$
to find the lower bound of the product, work out $4.45 \times 6.35 = 28.2575$

For any product, the lower bound of the product is worked out using the lower bound of each number in the product. Similarly, for the upper bound, the upper bound of the product is worked out using the upper bound of each number in the product.

However, the lower bound of the difference 6.0 – 3.8, where both numbers are written correct to 1 decimal place, is **not** found by subtracting the lower bound of 3.8 from the lower bound of 6.0. The lower bound of the difference is the difference between the lower bound of 6.0 and the **upper** bound of 3.8

Lower bound $= 5.95 - 3.85 = 2.1$ (**not** $6.05 - 3.85 = 2.2$)

Example 7

Correct to 1 decimal place, $x = 4.8$ and $y = 2.4$

Work out the lower bounds of

a xy **b** $x - y$ **c** $x + y$ **d** $\frac{x}{y}$

Solution 7

a $\square \times \square$

b $\square - \square$

c $\square + \square$

d $\square \div \square$

Example 8

$H = \frac{v^2}{2g}$ is a formula used to find the height H, of a stone thrown upwards at a speed v.

$v = 10$ correct to the nearest integer, $g = 9.8$ correct to 2 significant figures.

a Write down the upper bound of g.

b Work out the lower bound of H.

Give your answer correct to 3 decimal places.

Solution 8

a Upper bound of $g = \square$

b Lower bound of $H = \frac{\square^2}{2 \times \square}$

$= 4.5812...$

$= 4.581$

An answer correct to 3 decimal places is required.

Topics in Arithmetic **Upper and Lower Bounds** DATE:_____

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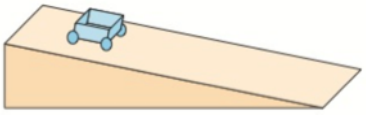
Example: In a science experiment, a trolley travelled 5.6 meters in 3.44 seconds. Calculate the average speed of the trolley.



Measuring instrument

Work out the upper bound for the speed. Round to 3 s.f.

$$\frac{5.65}{3.435} = 1.64\ m/s$$

Work out the lower bound for the speed. Round to 3 s.f.

$$\frac{5.55}{3.445} = 1.61\ m/s$$

When carrying out calculations with numbers that have been rounded it is possible to calculate the range of answers that can be produced depending on the accuracy to which the numbers have been given.

If a number has been written as 5.6 correct to one decimal place then the true value of the number lies between 5.55 and 5.65 The value at the lower boundary of the interval is called the **lower bound**. The value at the upper boundary is called the **upper bound**. In this case, 5.55 is the lower bound and 5.65 is the upper bound.

If a number has been written as 3.44 correct to 3 significant figures, then the true value of the number lies between 3.435 to 3.445 so the lower bound is 3.435 and the upper bound is 3.445

Example 6

Write down **i** the lower bound **ii** the upper bound of

a 64 correct to 2 significant figures **b** 32.8 correct to 1 decimal place.

Solution 6

a **i** lower bound = 63.5
ii upper bound = 64.5

63 63.5 64 64.5 65

Numbers in the red interval from 63.5 to 64.5 are closer to 64 than to 63 or to 65 The lower bound (the value at the lower boundary of this interval) is 63.5 The upper bound (the value at the upper boundary of the interval) is 64.5

b **i** lower bound = 32.75
ii upper bound = 32.85

32.7 32.75 32.8 32.85 32.9

Numbers in the red interval from 32.75 to 32.85 are closer to 32.8 than to 32.7 or to 32.9

If numbers in calculations have been rounded, the final answer will not be exact. It is, however, possible to find the lower bound and the upper bound of the answer.

For example, in the product 4.5×6.4 where both numbers have been rounded to 1 decimal place

to find the upper bound of the product, work out $4.55 \times 6.45 = 29.3475$
to find the lower bound of the product, work out $4.45 \times 6.35 = 28.2575$

For any product, the lower bound of the product is worked out using the lower bound of each number in the product. Similarly, for the upper bound, the upper bound of the product is worked out using the upper bound of each number in the product.

However, the lower bound of the difference 6.0 – 3.8, where both numbers are written correct to 1 decimal place, is **not** found by subtracting the lower bound of 3.8 from the lower bound of 6.0. The lower bound of the difference is the difference between the lower bound of 6.0 and the **upper** bound of 3.8

Lower bound $= 5.95 - 3.85 = 2.1$ (**not** $6.05 - 3.85 = 2.2$)

Example 7

Correct to 1 decimal place, $x = 4.8$ and $y = 2.4$

Work out the lower bounds of

a xy **b** $x - y$ **c** $x + y$ **d** $\frac{x}{y}$

Solution 7

a $\boxed{4.75} \times \boxed{2.35}$
$= 11.1625$
Lower bound × Lower bound

b $\boxed{4.75} - \boxed{2.45}$
$= 2.3$
Lower bound − Upper bound

c $\boxed{4.75} + \boxed{2.35}$
$= 7.1$
Lower bound + Lower bound

d $\boxed{4.75} \div \boxed{2.45}$
$= 1.938\,775\,5$
Lower bound ÷ Upper bound

Example 8

$H = \frac{v^2}{2g}$ is a formula used to find the height H, of a stone thrown upwards at a speed v.

$v = 10$ correct to the nearest integer, $g = 9.8$ correct to 2 significant figures.

a Write down the upper bound of g.

b Work out the lower bound of H.

Give your answer correct to 3 decimal places.

Solution 8

a Upper bound of $g = 9.85$

b Lower bound of $H = \frac{9.5^2}{2 \times 9.85}$
Lower bound of $v^2 \div (2 \times$ Upper bound of $g)$
$= 4.5812...$
$= 4.581$
An answer correct to 3 decimal places is required.

Topics in Arithmetic **Upper and Lower Bounds** DATE:_____

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You can use your calculator.

1 $p = 18$ correct to the nearest integer, $q = 12$ correct to the nearest integer.

a Write down the lower bound of **i** p **ii** q

b Write down the upper bound of **i** p **ii** q

c Work out the lower bound of **i** $p+q$ **ii** $p-q$ **iii** pq **iv** $\frac{p}{q}$

d Work out the upper bound of **i** $p+q$ **ii** $p-q$ **iii** pq **iv** $\frac{p}{q}$

2 $r = 16.4$ correct to 1 decimal place, $t = 4.7$ correct to 1 decimal place.

a Work out the lower bound of **i** $r+t$ **ii** $r-t$ **iii** rt **iv** $\frac{r}{t}$

b Work out the upper bound of **i** $r+t$ **ii** $r-t$ **iii** rt **iv** $\frac{r}{t}$

3 $x = 6.4$ correct to 1 decimal place, $y = 8.3$ correct to 1 decimal place.

a Work out the lower bound of **i** $x+y$ **ii** $y-x$ **iii** xy **iv** $\frac{y}{x}$

b Work out the upper bound of **i** $x+y$ **ii** $y-x$ **iii** xy **iv** $\frac{y}{x}$

4 $k = 2.45$ correct to 2 decimal places.

a Work out the lower bound of **i** $4k$ **ii** $\frac{1}{k}$ **iii** k^2 **iv** $\sqrt{k}$

b Work out the upper bound of **i** $4k$ **ii** $\frac{1}{k}$ **iii** k^2 **iv** $\sqrt{k}$

5 $x = 4.62$ correct to 2 decimal places, $y = 2.5$ correct to 1 decimal place.

a Work out the lower bound of **i** $x+y$ **ii** $x-y$ **iii** xy **iv** $\frac{y}{x}$

b Work out the upper bound of **i** $x+y$ **ii** $x-y$ **iii** xy **iv** $\frac{y}{x}$

ANSWERS

1 a	**i** 17.5	**ii** 11.5		
b	**i** 18.5	**ii** 12.5		
c	**i** 29	**ii** 5	**iii** 201.25	**iv** 1.4
d	**i** 31	**ii** 7	**iii** 231.25	**iv** 1.609
2 a	**i** 21	**ii** 11.6	**iii** 76.0275	**iv** 3.442
b	**i** 21.2	**ii** 11.8	**iii** 78.1375	**iv** 3.5376
3 a	**i** 14.6	**ii** 1.8	**iii** 52.3875	**iv** 1.2791
b	**i** 14.8	**ii** 2.0	**iii** 53.8575	**iv** 1.3150
4 a	**i** 9.78	**ii** 0.4073	**iii** 5.978025	**iv** 1.5652
b	**i** 9.82	**ii** 0.4090	**iii** 6.027025	**iv** 1.5668
5 a	**i** 7.065	**ii** 2.065	**iii** 11.30675	**iv** 0.52973
b	**i** 7.175	**ii** 2.175	**iii** 11.79375	**iv** 0.55255

6 $p = 3.8$ correct to 1 decimal place, $q = 4.60$ correct to 2 decimal places.

a Work out the lower bound of **i** p^2 **ii** $3p + 2q$ **iii** $\dfrac{p+q}{p}$

b Work out the upper bound of **i** p^2 **ii** $3p + 2q$ **iii** $\dfrac{p+q}{p}$

7 Scott cycles at a steady speed of 10 metres per second correct to the nearest metre per second.

a Work out the lower bound of the time Scott takes to cycle exactly 100 metres.

b Work out the upper bound of the time Scott takes to cycle exactly 100 metres.

8 The length of a rectangle is 16 cm. The width of the rectangle is 12 cm. Both measurements have been given correct to the nearest cm.

a Find the lower bound of the perimeter.

b Find the upper bound of the perimeter.

c Find the lower bound of the area.

d Find the upper bound of the area.

9 $h = \dfrac{v^2}{2g}$, $v = 10.2$ correct to 3 significant figures. $g = 9.8$ correct to 2 significant figures.

Work out the difference between the lower bound of h and the upper bound of h.

10 The population of a country is 14.6 million correct to 3 significant figures.
8.5 million, correct to 2 significant figures, of the population are under 30 years of age.
Calculate the difference between the lower bound and the upper bound of the percentage of the population which are under 30 years of age.

ANSWERS

6 a i 14.0625 **ii** 20.44 **iii** 1.8142
b i 14.8225 **ii** 20.76 **iii** 1.83786

7 a 9.524 s
b 10.536 s

8 a 54 cm
b 58 cm
c 178.25 cm^2
d 206.25 cm^2

9 0.1583

10 1.08%